OPEN ACCESS Freely available online

Journal of Economic Intelligence and Mathematics

Review Article

Diagonal Condition in Multiplication Table of Z[i]/(α)

Chadaphorn Kodsueb*

School of Mathematical Sciences and Geoinformatics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, Thailand 30000.

ABSTRACT

Multiplication table of a ring with identity 1 is said to have the diagonal condition if 1's occur only on the main diagonal. In this paper, we study the diagonal condition in the ring of Gaussian integers Z[i]. Furthermore, we also find the Gaussian integers α so that the rings of Gaussian integers modulo α satisfy the diagonal condition.

Keywords: Diagonal Condition; Multiplication Table; Gaussian Integers; Quotient ring 2020 MSC: 11R04; 11T30; 11N13;

INTRODUCTION

In [1] S.K. Chebolu (2012), he observed that 1's only appear on the main diagonal of multiplication tables of $\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4$ and $\mathbb{Z}_8$ but not for $\mathbb{Z}_5$. The aim of [1] S.K. Chebolu (2012) is to find which positive integers n that make 1's occur only on the main diagonal (not off diagonal) of multiplication table of $\mathbb{Z}_n$. Given $\mathbb{Z}_n = \{1,2,\ldots,n-1\}$. Any ring which has 1's only appear on the main diagonal of its multiplication table. One says the ring satisfies diagonal condition, summarized in the following.

Galley Proof

Theorem: Let n be a positive integer for $\mathbb{Z}_n$. The statements are equivalent:

1. 1's occur only on the main diagonal (not off diagonal) of multiplication table of $\mathbb{Z}_n$.
2. If a is an element which has multiplicative inverse in $\mathbb{Z}_n$, then $a^2 = 1 \in \mathbb{Z}_n$.
3. If a is an integer relatively prime with n, then $n|a^2 - 1$.
4. If p is a prime such that $p \nmid n$, then $n|p^2 - 1$.

Finally, he found that 1's occur only on the main diagonal of multiplication table of $\mathbb{Z}_n$ if and only if n is a divisor of 24. This became the main theorem in [2] S.K. Chebolu (2012). He also provided five distinct proofs for this theorem. However, the special property about divisors of 24 has not ended there. In 2024, *J. C.* Lagarias proposed problems on Floor Quotients in American Math. Monthly problem 11747, Vol. 121, *p.* 83. His solution appears in Vol. 123 (2016), pp. 199–200 [9].

***Correspondence to:** Chadaphorn Kodsueb, School of Mathematical Sciences and Geoinformatics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, Thailand 30000. E-mail: Chadaphorn.Ko@sut.ac.th

Received: October 09, 2026; **Manuscript No:** JEIM-26-1036; **Editor Assigned:** October 26, 2026; Pre Qc No: JEIM-26-1036 (PQ); **Reviewed:** October 30, 2026; **Revised:** October 31, 2026; **Accepted:** November 02, 2026; **Published:** November 07, 2026

Citation: Kodsueb C (2026) Diagonal Condition in Multiplication Table of Z[i]/(α). J. Econ. Intell. Math. Health Res, Vol.1 Iss.1, November (2026), pp:77-84.

Copyright: © 2026 Chadaphorn Kodsueb. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Later, [3] Chebolu, S.K. and Mayers (2013) investigated on the (same) diagonal condition of multiplication table but in a different ring, $\mathbb{Z}_n[x]$. They had very interest result, and then formed it as a main theorem "For any positive integer m, the multiplication table for the polynomial ring $\mathbb{Z}_n[x_1, x_2, \cdots, x_m]$ has 1's only on the diagonal if and only if n is a divisor of 12". Another result involved with number 12 has shown in [17] Poonen, B. and Rodriguez-Villegas, F. (2000). They discussed the number 12 related to convex lattice polygons with one interior lattice point.

Additionally, there is one paper linked to both numbers 12 and 24. In [14] Godinho, L., von Heymann, F. and Sabatini, S. (2017), they were interested in connection with mirror-symmetry and Calabi-Yau toric varieties. This kind of thing leads into aspects of Topology and Geometry.

Beyond than that [11] Genzlinger, K. and Lockridge, K. (2015) studied the diagonal condition on multiplication cube of $\mathbb{Z}_n$. They gained the main theorem as "All 1's in the multiplication cube for $\mathbb{Z}_n$ lie exclusively on the diagonal or the coordinate planes (where one of the three coordinates is 1) if and only if n is a divisor of 4 or 6.

Nevertheless, the author'$saim$ is still about the diagonal condition in multiplication table of (another) classical ring: Gaussian integers $\mathbb{Z}[i]$, specifically a ring of Gaussian integers modulo α.

GUSSIAN INTEGERS

This section provides crucial background of Gaussian integers $\mathbb{Z}[i]$ and Gaussian integers modulo α where α is nonzero in $\mathbb{Z}[i]$. Some materials in this section, we take it partially from [3]K. Conrad (2016), [4]$D.S$. Dummit and $R.M$ Foote (2004) and [6]J.A. Gallian (2021).

Galley Proof

Basic knowledge of $\mathbb{Z}[i]$

Gaussian integers are complex numbers is in the form $a + bi$ where $a, b \in \mathbb{Z}$, denoted $\mathbb{Z}[i]$. In fact, $\mathbb{Z}[i]$ is subring of $\mathbb{C}$ with 1 is multiplicative identity. Since $\mathbb{C}$ is a field, $\mathbb{Z}[i]$ is an integral domain.

Definition For $\alpha = a + bi$, Norm of α denote by $\mathcal{N}(\alpha) := \alpha\bar{\alpha} = (a + bi)(a - bi) = \alpha^2 + \beta^2$.

Remark It is obvious that $\mathcal{N}(\alpha) \in \mathbb{N}_0 = \mathbb{Z}^+ \cup \{0\}$ if including the case $\alpha = 0$.

Theorem For $\alpha, \beta \in \mathbb{Z}[i], \mathcal{N}(\alpha\beta) = \mathcal{N}(\alpha)\mathcal{N}(\beta)$.

Definition The Gaussian integer β is a unit ifβ has multiplicative inverse in$\mathbb{Z}[i]i.e.$ there exists $\gamma \in \mathbb{Z}[i]$ such that $\gamma\beta = 1$.

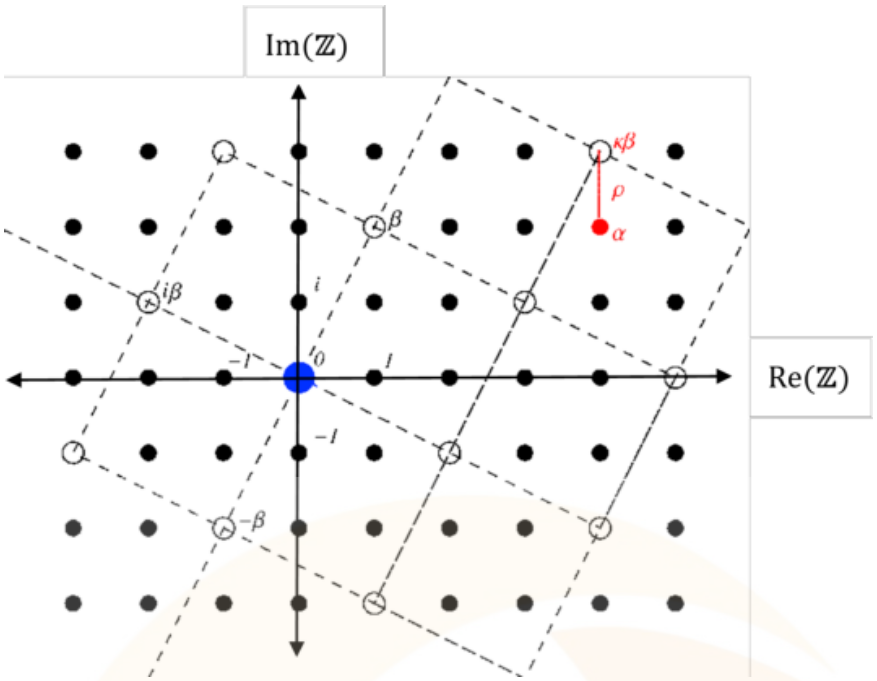
Theorem The only Gaussian integers that are units in $\mathbb{Z}[i]$ are $1, -1, i, -i$.

Definition For $\alpha, \beta \in \mathbb{Z}[i]$ where $\beta \neq 0$, we say that $\beta|\alpha$ if $\alpha = \beta\gamma$ for some$\gamma \in \mathbb{Z}[i]$. Then β is called a divisor of α.

Theorem For all α, β in $\mathbb{Z}[i]$, if $\beta|\alpha$ in $\mathbb{Z}[i]$ then $\mathcal{N}(\beta)|\mathcal{N}(\alpha)$ in $\mathbb{Z}$.

Theorem Division Algorithm in $\mathbb{Z}[i]$ Given $\alpha, \beta \in \mathbb{Z}[i]$ with $\beta \neq 0$. There exist κ, ρ in $\mathbb{Z}[i]$ such that $\alpha = \beta\kappa + \rho$ and $\mathcal{N}(\rho) < \mathcal{N}(\beta)$. In fact, we can choose ρ that makes $\mathcal{N}(\rho) < \frac{1}{}\mathcal{N}(\beta)$.

Proof. We provide proof without words in Geometry: Division Algorithm in $\mathbb{Z}[i]$.



From the figure, vector from 0 to $i\beta$ is perpendicular with vector from 0 to β in Complex Plane $\mathbb{C} = \mathbb{R}^2$. So the set $\beta\mathbb{Z}[i] = \{\beta\kappa \mid \beta \in \mathbb{Z}[i]\}$ forms squares with length $|\beta| = \sqrt{\mathcal{N}(\beta)}$ netting as lattice. It is easy to see that α appearing inside at least 1 square. Let $\beta\kappa$ be the most nearest corner point with point α. Given $\rho = \alpha - \beta\kappa \in \mathbb{Z}[i]$. Then $|\rho|$ is less than or equal to half a diagonal length, i.e. $\sqrt{\mathcal{N}(\rho)} = |\rho| \leq \frac{\sqrt{2}}{2}|\beta| < \sqrt{\mathcal{N}(\beta)}$.

Known fact is $\mathbb{Z}[i]$ is a Euclidean Domain (ED). Then$\mathbb{Z}[i]$ is also a PID and a UFD.

Definition Let $\gamma \in \mathbb{Z}[i]$, we call $1, -1, i, -i, \gamma, -\gamma, \gamma i, -\gamma i$ are trivial factors ofγ.

Definition For $\alpha \in \mathbb{Z}[i]$ which is not a unit, α is a factorized element (reducible element). If α is not a trivial factor and α has only trivial factors, then α is a irreducible element or called a Gaussian prime.

Theorem If $\mathcal{N}(\alpha)$ is a prime in $\mathbb{Z}$, then α is a Gaussian prime in$\mathbb{Z}[i]$.

Definition A common divisor of α and β is a Gaussian integer δ such that $\delta|\alpha$ and $\delta|\beta$. Denote $\delta := \gcd(\alpha, \beta)a$ greatest common divisor of α and β if and only if

1. $\delta|\alpha$ and $\delta|\beta$,
2. For all $\gamma \in \mathbb{Z}[i]$, if $\gamma|\alpha$ and $\gamma|\beta$ then $\gamma|\delta$. One says α and β are relatively (Gaussian) primes if $\delta = \gcd(\alpha, \beta)$ is a unit in $\mathbb{Z}[i]$.

Theorem Given π is a Gaussian prime. For all $\alpha, \beta \in \mathbb{Z}[i]$, if $\pi|\alpha\beta$ then $\pi|\alpha$ or $\pi|\beta$.

Remark Since $\mathbb{Z}[i]$ is a Unique Factorization Domain, Gaussian integers which are nonzero and non-unit can be factorized as multiplication of distinct Gaussian primes and a unit. § Author's explanation

Coset representatives of quotient ring $\mathbb{Z}[i]/(\alpha)$

This subsection gives concepts of congruence–modulo and coset representatives in $\mathbb{Z}[i]/(\alpha)$.

Definition For Gaussian integers α, β and γ, we say that β congruence γ modulo α denoted by $\beta \equiv \gamma \ (mod\ \alpha)$, when $\alpha|(\beta - \gamma)$.

Remark It can be proved that Congruence–Modulo is an equivalence relation on $\mathbb{Z}[i]$. The equivalence classes are cosets in quotient ring $\mathbb{Z}[i]/(\alpha)$. Next, we provide methods to find coset representatives of $\mathbb{Z}[i]/(\alpha)$ when $\alpha = a + bi$ through multiples of Gaussian in grid as followed.

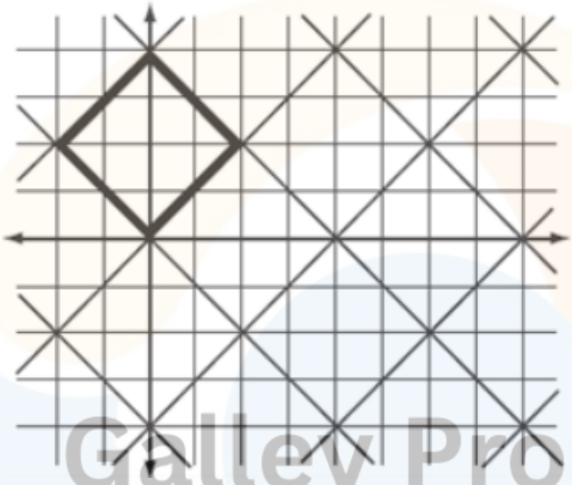
Let $m, n \in \mathbb{Z}$. Consider $(a + bi)(m + ni) = (a + bi)m + (-b + ai)^n$.

Plot points (a, b) and $(-b, a)$ in the grid, then form two vectors from origin to these points.

Extend the sided of two vectors to create a (main) square. Coset representatives of $\mathbb{Z}[i]/(\alpha)$ are points located inside the square and on two main vectors including origin.

Translate the square till it can be filled the grid. Then, we have (all) cosets.

Examples 2.2.2 Find coset representatives of $\mathbb{Z}[i]/(2 + 2i)$. First, consider $(2 + 2i)(m + ni) = (2 + 2i)m + (-2 + 2i)^n$. Then follow steps 2. and 3. :



Galley Proof

From the figure, the coset representatives of $\mathbb{Z}[i]/(2 + 2i)$ are $0, i, 2i, 3i, 1 + i, 1 + 2i, -1 + i, -1 + 2i$ equivalent with points $(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (-1,1), (-1,2)$ in grid.

Lemma For $a, b, c, d, m \in \mathbb{Z}$, we have

$a \equiv b (mod\, c)$ in $\mathbb{Z}$ if and only if $a \equiv b (mod\, c)$ in $\mathbb{Z}[i]$.

$a + bi \equiv c + di (mod\, m)$ if and only if $a \equiv c (mod\, m)$ and $b \equiv d (mod\, m)$.

Let $\mathfrak{N}(\alpha)$ be the number of coset representatives of $\mathbb{Z}[i]/(\alpha) i.e.\ \mathfrak{N}(\alpha) := |\mathbb{Z}[i]/(\alpha)|, \alpha \in \mathbb{Z}[i]$.

Lemma If $m \in \mathbb{Z}$ is nonzero, then $\mathfrak{N}(m) = m^2$.

Lemma If $\alpha \neq 0$ in $\mathbb{Z}[i]$, then $\mathfrak{N}(\bar{\alpha}) = \mathfrak{N}(\alpha)$.

Lemma If α and β are nonzero in $\mathbb{Z}[i]$, then $\mathfrak{N}(\alpha\beta) = \mathfrak{N}(\alpha)\mathfrak{N}(\beta)$. In another word, function $\mathfrak{N}$ is multiplicative.

Theorem If $\alpha \neq 0$ in $\mathbb{Z}[i]$, then $\mathfrak{N}(\alpha) = \mathcal{N}(\alpha)$.

Proof. By Lemma 2.2.6, $\mathfrak{N}(\alpha\bar{\alpha}) = \mathfrak{N}(\alpha)\mathfrak{N}(\bar{\alpha})$. From Lemma 2.2.5, we have the right hand side of previous equation is $\mathfrak{N}(\alpha)^2$. Since $\alpha\bar{\alpha} = \mathfrak{N}(\alpha)$ is integer and Lemma 2.2.4, it can be concluded that $\mathfrak{N}(\alpha\bar{\alpha}) = \mathcal{N}(\alpha)^2$. Thus, $\mathcal{N}(\alpha)^2 = \mathfrak{N}(\alpha)^2$. Hence, we have $\mathcal{N}(\alpha) = \mathfrak{N}(\alpha)$.

Gaussian primes

In this subsection, we find all Gaussian primes from Gaussian integers which divide primes in

$\mathbb{Z}$. Definition In $\mathbb{Z}[i]$, we say that $\alpha \sim \beta$ if $\alpha = u\beta$ or $\alpha = u\beta$ where u is a unit in $\mathbb{Z}[i]$.

Lemma 2.3.1 Let π be a Gaussian prime. Then, there exists a prime$p \in \mathbb{Z}$ such that $\pi|p$.

We can find all Gaussian primes from Gaussian integers which divide primes in $\mathbb{Z}[i]$.

Theorem A prime in $\mathbb{Z}$ is a factorized element (reducible element) in $\mathbb{Z}[i]$ if and only if p can be written as a sum of two squares.

Remark Any primes $p \in \mathbb{Z}$ which cannot be written as a sum of two squares, then p is not a factorized element (reducible element). Thus, p is a Gaussian primes.

Corollary If p is a prime in $\mathbb{Z}$ such that $p \equiv 3(mod\ 4)$, then p is a Gaussian prime.

Lemma Given $p \in \mathbb{Z}$ is a prime. Either$p = 2$ or $p \equiv 1(mod\ 4)$ if and only if there exist integersa, b such that $p = a^2 + b^2$.

Theorem Let p be a prime in $\mathbb{Z}$ such that $p \equiv 1(mod\ 4)$. Then $p = \pi\bar{\pi}$ where $\pi = a + bi$ and $\bar{\pi} = a - bi$ whilst $\pi \nsim \bar{\pi}$.

Proof. By, there exist integers a, b such that $p = a^2 + b^2$. So, $p = \pi\bar{\pi}$ where $\pi = a + bi$ and $\bar{\pi} = a - bi$. Assume that $a + bi \sim a - bi$. Then, there are 4 cases as followed

• case $a + bi = a - bi$: we have $b = 0$. Then $p = a^2, a$ contradiction.

• case $a + bi = -(a - bi)$: we have $a = 0$. Then $p = b^2, a$ contradiction.

• case $a + bi = i(a - bi)$: we get $a = b$. Then $p = 2a^2 = 2b^2, a$ contradiction.

• case $a + bi = -i(a - bi)$: we get $a = -b$. Then $p = 2b^2 = 2a^2, a$ contradiction. Thus, $\pi = a + bi \nsim a - bi = \bar{\pi}$.

Theorem These are all Gaussian primes:

1. $1 + i$,
2. p where p are primes in $\mathbb{Z}$ s.t. $p \equiv 3(mod\ 4)$,
3. $\pi = a + bi$ and $\bar{\pi} = a - bi$ when $\mathcal{N}(\pi) = \mathcal{N}(\bar{\pi}) = a^2 + b^2 = p$, and p are primes in $\mathbb{Z}$ s.t. $p \equiv 1\ (mod\ 4)$.

Proof. We can find all Gaussian primes which divide prime $p \in \mathbb{Z}$ from Lemma 2.3.1:

1. When $p = 2$, we have $2 = (1 + i)(1 - i) = -i(1 + i)^2$. Then $1 + i$ is satisfied the condition i.e. $1 + i|2$ and $1 + i \nsim 1 - i$, ¶ Main theorem for this section.
2. When $p \equiv 3(mod\ 4), p$ is a Gaussian prime by Corollary 2.3.4,
3. When $p \equiv 1(mod\ 4)$, by Theorem 2.3.6, π and $\bar{\pi}$ are Gaussian primes where $p = \pi\bar{\pi}$ and $\pi \nsim \bar{\pi}$.

Thus, we have proved.

MAIN RESULTS

From subsection 2.3 (Theorem 2.3.7), we know that all Gaussian primes are in the form

1. $1 + i$,
2. p where p are primes in $\mathbb{Z}$ s.t. $p \equiv 3(mod\ 4)$,
3. $\pi = a + bi$ and $\bar{\pi} = a - bi$ when $\mathcal{N}(\pi) = \mathcal{N}(\bar{\pi}) = a^2 + b^2 = p$, and p are primes in $\mathbb{Z}$ s.t. $p \equiv 1\ (mod\ 4)$.

We also know that any Gaussian integer $\alpha \neq 0$ and α not a unit can be written as Gaussian primes factorization in the following theorem.

Galley Proof

Theorem If $\alpha = u\pi_1^{c_1}\pi_2^{c_2}\cdots\pi_r^{c_r}$ where u is unit and π_i are Gaussian primes such that $u \nsim \pi_i$, then

$\mathbb{Z}[i]/(\alpha) \cong \mathbb{Z}[i]/(u) \times \mathbb{Z}[i]/(\pi_1^{c_1}) \times \mathbb{Z}[i]/(\pi_2^{c_2}) \times \cdots \times \mathbb{Z}[i]/(\pi_r^{c_r})$.

Proof. This is trivial from Chinese Remainder Theorem, see statement in [9]T. Koshy (2002).

Theorem $\mathbb{Z}[i]/((1+i)^n)$ do not satisfy diagonal condition when $n \geq 3$.

Proof. Suppose that $\mathbb{Z}[i]/((1+i)^n)$ satisfy diagonal condition for some $n \geq 3$. Then, for all $\alpha \in \mathbb{Z}[i]$ s.t. $\gcd(\alpha, 1+i) = \delta$ where δ is unit, we have $\alpha^2 \equiv 1(mod\,(1+i)^n)$. In $(\mathbb{Z}[i]/(1+i)^3)^\times$, we get $(-2+i)^2 = 4-4i-1 \equiv -1(mod\,(1+i)^3)$ that makes $(-2+i)^4 \equiv 1(mod\,(1+i)^3)$. Since $\mathbb{Z}[i]/((1+i)^n)$ satisfy diagonal condition for some $n \geq 3$, $(-2+i)^2 \equiv 1(mod\,(1+i)^n)$ and hence $(-2+i)^2 \equiv 1\ (mod\,(1+i)^3)$. Thus, $-1 \equiv 1(mod\,(1+i)^3)$ so that $(1+i)^3 | -2$. This leads a contradiction, the statement we supposed is wrong. Hence, $\mathbb{Z}[i]/((1+i)^n)$ do not satisfy diagonal condition for all $n \geq 3$.

Remark If the order of Gaussian integers modulo α is greater than or equal to 3, then multiplication table of $\mathbb{Z}[i]/(\alpha)$ do not have diagonal condition.

Lemma $\mathbb{Z}[i]/(p)$ do not satisfy diagonal condition when p are primes in $\mathbb{Z}$ s.t. $p \equiv 3(mod\,4)$.

Proof. Since p are primes in $\mathbb{Z}$ such that $p \equiv 3(mod\,4)$, p are Gaussian primes. Then $\mathbb{Z}[i]/(p)$ is a finite field which has order p^2 because (p) is the maximal ideal. Thus, $(\mathbb{Z}[i]/(p))^\times$ is a cyclic group with its order is equal to $p^2 - 1 \geq 3$. By Remark 3.3, the statement of lemma is proved.

Theorem $\mathbb{Z}[i]/((p)^n)$ do not satisfy diagonal condition when $n \geq 2$ and p are primes in $\mathbb{Z}$ such that $p \equiv 3(mod\,4)$.

Galley Proof

Proof. Assume that $\mathbb{Z}[i]/((p)^n)$ satisfy diagonal condition for some $n \geq 2$. Then, we have $\forall \alpha \in \mathbb{Z}[i]$ s.t. $\gcd(\alpha, p) = \delta$ where δ is unit, $\alpha^2 \equiv 1(mod\ pn)$. However, there exists $\beta \in \mathbb{Z}[i]/(p)$ such that the order of β modulo p is $p^2 - 1 \geq 3$. That makes $\beta^2 \not\equiv 1(mod\,p)$, contradicts with $\beta^2 \equiv 1(mod\,p)$ by assumption. Hence, the statement of theorem is confirmed.

Lemma $\mathbb{Z}[i]/(\pi)$ and $\mathbb{Z}[i]/(\bar{\pi})$ do not satisfy diagonal condition when $\pi\bar{\pi} = p$ and p are primes in $\mathbb{Z}$ such that $p \equiv 1(mod\,4)$.

Proof. By Theorem 2.3.7, $\pi \nsim \bar{\pi}$ without loss of generality. Since π is a Gaussian prime, $\mathbb{Z}[i]/(\pi)$ is a field because (π) is a maximal ideal. Thus, $(\mathbb{Z}[i]/(\pi))^\times$ is a cyclic group with order $p-1$. Then, there exists $\alpha \in \mathbb{Z}[i]$ such that the order of α modulo π is equal to $p - 1 \geq 3$ so that $\alpha^2 \not\equiv 1(mod\,\pi)$. Hence, $\mathbb{Z}[i]/(\pi)$ does not satisfy diagonal condition.

Theorem $\mathbb{Z}[i]/((\pi)^n)$ and $\mathbb{Z}[i]/((\bar{\pi})^n)$ do not satisfy diagonal condition when $n \geq 2$ and $\pi\bar{\pi} = p$ where p are primes in $\mathbb{Z}$ such that $p \equiv 1(mod\,4)$.

Proof. By Theorem 2.3.7, $\pi \nsim \bar{\pi}$ without loss of generality. Suppose $\mathbb{Z}[i]/((\pi)^n)$ satisfy diagonal condition. Then, $\alpha^2 \equiv 1(mod\,(\pi)^n)$ for all $\alpha \in \mathbb{Z}[i]$. In $\mathbb{Z}[i]/(\pi)$, there exists $\beta \in \mathbb{Z}[i]$ such that the order of β modulo π is $p \geq 3$ so that $\beta^2 \not\equiv 1(mod\,\pi)$. By the assumption, we need to have $\beta^2 \equiv 1\ (mod\,(\pi)^n)$, a contradiction. Thus, the statement of theorem is proved.

From Theorem 3.1 to Theorem 3.7 including remark and lemmas, we can generate the main theorem as followed:

Theorem Multiplication table of $\mathbb{Z}[i]/(\alpha)$ have diagonal condition if and only if $\alpha = 1 + i$ or $\alpha = (1 + i)^2$.

OPEN PROBLEMS AND INTERESTING POINTS

The author suggests readers to investigate diagonal condition on several interesting rings e.g. $\mathbb{Q}[i]$, $\mathbb{Z}\left[\frac{1+\sqrt{(-3)}}{2}\right]$, Real/ Imaginary Quadratic fields ($\mathbb{Q}[\sqrt{D}]$ where D is square-free, Real when $D > 0$; Imaginary when $D < 0$) or any other imaginary quadratic fields have units only 1 and -1. † Also, all related open problems such as Floor Quotient for the previous-mentioned domains with a variant problem analogs with what the author did in this paper.

DISCLOSURE

The author declares that there are no conflicts of interest that could have influenced the objectivity of this research or the writing of this paper.

ACKNOWLEDGEMENTS

The author would like to thank her BS advisor Assoc. Prof. Dr. Ajchara Harnchoowong for inspiration on research of Number Theory. Additionally, the author would like to praise productive comments from Prof. Jeffrey Clark Lagarias while attending the 2026 Gainesville international Number Theory conference at University of Florida, USA. The most important part to mention is her family, they always cheer her up in any stage of this research work.

† This part is suggested by Prof. Jeffrey Clark Lagarias

Galley Proof

REFERENCES

1. Burns G, Alper AM, Nowick AS. Introduction to group theory with applications. New York: Academic Press; 1977 Jan 1.
2. Chebolu SK. What is special about the divisors of 24?. Mathematics Magazine. 2012 Dec 1;85(5):366-72.
3. Chebolu SK, Mayers M. What is special about the divisors of 12?. Mathematics Magazine. 2013 Apr 1;86(2):143-6.
4. Cohen S. A study of Gaussian integers.
5. Kodsueb C. Diagonal Condition in Multiplication Table of $\displaystyle {\,\mathbb {Z}[i]/(\alpha)} $. arXiv preprint arXiv:2606.02975. 2026 Jun 2.
6. Davenport H. Multiplicative number theory. Springer Science & Business Media; 2013 Jun 29.
7. Dresden G, Dymàček WM. Finding factors of factor rings over the Gaussian integers. The American Mathematical Monthly. 2005 Aug 1;112(7): 602-11.
8. Gallian J. Contemporary abstract algebra. Chapman and Hall/CRC; 2021 Jan 19.
9. Genzlinger K, Lockridge K. Sophie Germain primes and involutions of $\mathbb{Z}_n^\times$. Involve, a Journal of Mathematics. 2015 Jun 23;8(4):653-63.
10. Gerjets MS. The distribution of primitive roots in fields of order p^2.
11. Ginzburg A. Representation of groups by generalized normal multiplication tables. Canadian Journal of Mathematics. 1967 Jan;19:774-91.
12. Godinho L, von Heymann F, Sabatini S. 12, 24 and beyond. Advances in Mathematics. 2017 Oct 15;319:472-521.
13. Koshy T. Elementary number theory with applications. Academic press; 2002.
14. Lagarias JC, Richman DH. The family of a-floor quotient partial orders. InCombinatorial and Additive Number Theory, New York Number Theory Seminar 2022 May 24 (pp. 283-318). Cham: Springer Nature Switzerland.
15. Poonen B, Rodriguez-Villegas F. Lattice polygons and the number 12. The American Mathematical Monthly. 2000 Mar 1;107(3):238-50.
16. Sabuncu C, Táfula C. The multiplication table problem in large dimensions. arXiv preprint arXiv:2608.16163. 2026 Aug 17.
17. Schlitt J. Multiplication Tables for Integers with Restricted Prime Factors. arXiv preprint arXiv:2603.19212. 2026 Mar 19.
18. Wahab MA. Elements of Group Theory and Multiplication Tables. InSymmetry Representations of Molecular Vibrations 2022 Sep 21 (pp. 49-122). Singapore: Springer Nature Singapore.

Galley Proof